A formulation for frictionless contact using a material model and high order finite elements

Background
Contact problems in solid mechanics are classically solved by the $h$-version of the finite element method [1]. The constraints are enforced along a priori defined interfaces on the surfaces of elastic bodies under consideration.

Contact material
We present a novel approach to model frictionless contact using high order finite elements ($p$-FEM) [2]. Here, a specially designed material is used, which is inserted into regions surrounding contacting bodies [3]. Contact constraints are thus enforced on the same manifold as the accompanying structural problem. Our contact material model is based on the hyperelastic formulation by Hencky [4]:

$$W_I(J, A, 2A_2 A_3) = \mu \sum_{ii} \Delta_i^2 n_i^2 + \frac{\nu}{2} (n_1 n_2)^2,$$

where

$$J = A_1 A_2 A_3.$$

The material parameters $\mu$ and $\nu$ are scaled by a contact stiffness $c$ to regularize the Karush-Kuhn-Tucker conditions for normal contact:

$$g \geq 0 \quad \text{No normal penetration}$$

$$R \leq 0 \quad \text{Only compressive forces}$$

$$g \cdot R \quad \text{Consistency}.$$

The resulting principal stresses then read

$$\sigma_{ij} = \frac{c}{J} \left( 2 \mu \ln(J_i) + A_i \ln(J) \right).$$

2D model problem
The model problem under consideration is a slotted block subjected to a constant, vertical load. The physical part shown in grey contains a neo-Hookean material, whereas the slot is filled with the contact material model. Fillets at the corners of the slot are treated according to the finite cell method [5].

Numerical investigations showed, that modes inside the contact domain of order $p > 1$ might collapse. To overcome this problem, higher modes inside the contact domain are deactivated, while edge modes, on the interface to the physical domain remain active.

The influence of the contact stiffness $c$ on the resulting minimum gap $g_{\text{min}}$ is investigated for an ansatz order $p = 3$. The ratio of $g_{\text{min}}$ and the initial gap $g_0$ approaches zero as $c$ is reduced. The material, thus, converges to the limit state defined by the KKT conditions. Furthermore, the gap ratio for a contact stiffness of $c = 10^{-5}$ already lies in the range of 10%, which is sufficient for many engineering applications.

Conclusion
The proposed formulation works well for non-matching discretizations on adjacent contact interfaces and handles self contact naturally. Since the non-penetrating conditions are solved in a physically consistent manner, there is no need for an explicit contact search. By application of high order finite elements, structures can be discretized with only a few coarse finite elements. This allows the simulation of complex deformation scenarios with a lower number of degrees of freedom, compared the $h$-version of the FEM.

References