Multi-Level *hp*-Adaptivity: High-Order Mesh Adaptivity without the Difficulties of Constraining Hanging Nodes

Classical *hp*-Refinement Schemes

**Idea:** Coarse elements with a high error contribution are replaced by finer elements.

**Advantage:** High flexibility and possibility of automation

**Challenge:** Hanging nodes render the mesh irregular, as the shape functions between the interconnected elements are no longer C^0 continuous. These, therefore, require appropriate constraining.

The implementation of constraining is challenging, especially when multiply constrained modes have to be handled. Therefore, most *hp*-Finite Element codes restrict themselves to *1-irregular* meshes.

The *hp*-d-Refinement Scheme

**Idea:** Capture fine-scale solution characteristics by superposing a fine *h*-overlay mesh on a coarse, high-order base mesh in the domain of interest [1].

**Linear independence** is guaranteed by “deactivating” all direct descendants of base mesh nodes.

**Compatibility** is ensured by applying homogeneous Dirichlet boundary conditions on the overlay mesh.

**Advantage:** Hanging nodes are avoided by definition.

Hierarchical Extension

**Idea:** Hierarchically refine towards a singularity using multiple overlay meshes [2].

**Advantage:** Fast refinement towards singularity.

Multi-Level *hp*-Refinement

**Idea:** Use hierarchical, *high-order overlay meshes* to yield the full potential of *hp*-refinement schemes [3].

**Linear independence** is ensured by deactivating all topological components that have active sub-components.

**Compatibility** is ensured by homogeneous Dirichlet boundary conditions on the overlay meshes.

**Advantage:** Yield full *hp*-refinement capabilities with arbitrary hanging nodes at a comparably small implementational effort.

Numerical Examples

Non-Smooth Solution Benchmark

Result: Exponential convergence for non-smooth problems in the pre-asymptotic range.

Automatic Multi-Level *hp*-Refinement and Coarsening

**Idea:** Solve the wave equation and refine elements with high energy contribution *\|\hat{u}\|_H^2 > tol.*

References


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