

The Finite Cell Method for the Simulation of Additive Manufacturing

1 Overview

Additive manufacturing (AM) has emerged as a very promising technique for creating highly complex and customized solid structures on the basis of digital models. AM successively joins layers of material of different shapes to form a finished part. Numerous variants of this technique have been proposed over the past years. One special additive manufacturing process able to produce load bearing parts is selective laser melting (SLM). SLM fuses metal powder with a laser as illustrated in figure 1. In order to use parts produced by SLM e.g. in aerospace or orthopedics, the mechanical performance of the end product must be understood. To this end it is advantageous to simulate the SLM process in order to predict the dimensional accuracy and the residual thermal stresses within the part.

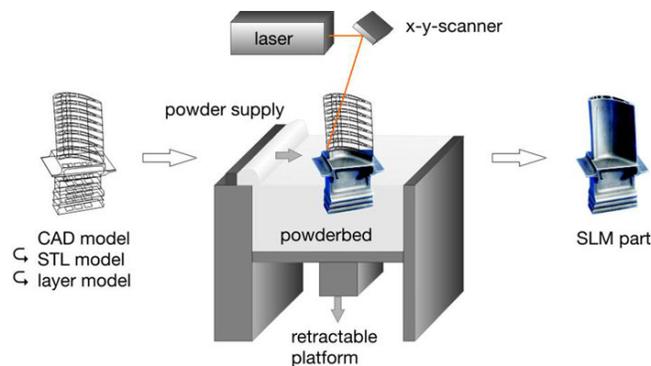


Figure 1: Selective laser melting process¹

There are numerous challenges involved in the simulation of the SLM process. The domain evolves during the process due to adding new layers of powder. Phase changes from powder to liquid to solid, which occur under the influence of the laser and states need to be tracked as a conversion is only possible from powder to liquid to solid but not from solid to powder.

Moreover it is a multi-scale process. The laser only has a diameter of approximately 100 micrometers i.e. it is highly localized but its effects are critical to the process such that this scale needs to be resolved. However, the scale of the finished product is in the order of decimeters and a clear separation is not straightforward because the scale of the layers which comprise a finished product lies in between. In addition to the spatial scales, the temporal scale differences

¹Xi'an Bright Laser Technologies LTD. [Online; accessed February 03, 2016]. URL: <http://www.xa-bltd.com/en/about/?116.html>

are large as well. Local melting and solidification only take milliseconds but the process of producing a part may take up to a few days.

All these challenges are posed on computational domains which are not only changing in time but also their shape and topology may be extremely complex.

In this contribution we propose to use the Finite Cell Method (FCM)² laid out in section 2, combined with the multi-level *hp*-method explained in section 3 as a basis for simulation of this transient multi-scale and multi-physics problem. We describe the mathematical and implementational structure for simulating the SLM process in section 4 and discuss computational results in section 5 before we conclude with section 6.

2 The Finite Cell Method

The Finite Cell Method combines the high-accuracy of high order finite elements with the flexibility of fictitious domain methods.

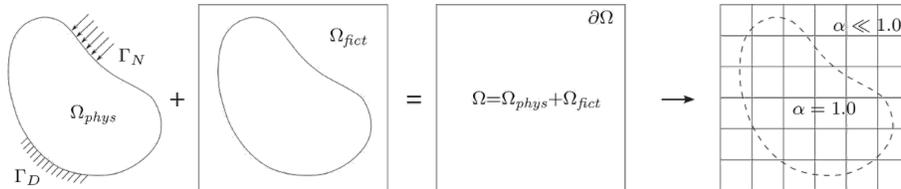


Figure 2: The Finite Cell Method: physical domain Ω_{phys} , fictitious domain Ω_{fict} and computational domain Ω

The basic idea of the FCM is depicted in figure 2: A physical domain Ω_{phys} is extended with a fictitious domain Ω_{fict} , such that their combination yields a simple-shaped computational domain Ω . The corresponding boundary value problem is then solved on Ω instead of Ω_{phys} . The advantage lies in the fact that Ω may easily be meshed and the resulting discretization does not have to be changed if Ω_{phys} is evolving in time. As a model problem a general bilinear form of the boundary value problem is given in equation (1).

$$\mathcal{B}(\mathbf{u}, \mathbf{v})_{\Omega} = \int_{\Omega} [\mathbf{L}\mathbf{v}]^T \alpha(\mathbf{x}) \mathbf{C} [\mathbf{L}\mathbf{u}] d\Omega \quad (1)$$

where $\alpha(\mathbf{x})$ is a indicator function defined as

$$\alpha(\mathbf{x}) = \begin{cases} 1 & \forall \mathbf{x} \in \Omega_{phys} \\ 0 & \forall \mathbf{x} \in \Omega_{fict}. \end{cases} \quad (2)$$

²A. Düster et al. “The finite cell method for three-dimensional problems of solid mechanics”. In: *Computer Methods in Applied Mechanics and Engineering* 197 (2008), pp. 3768–3782.

The indicator function recovers the original geometry but introduces a jump in the continuous integral of equation (1) which must be taken into account during spatial integration of bilinear form. To this end, we apply adaptive methods such as space trees, which are easy to implement for Cartesian grids. A two dimensional version of the integration scheme for a high order element matrix (called cell matrix in the context of FCM) of a cut cell with a quadtree is illustrated in figure 3.

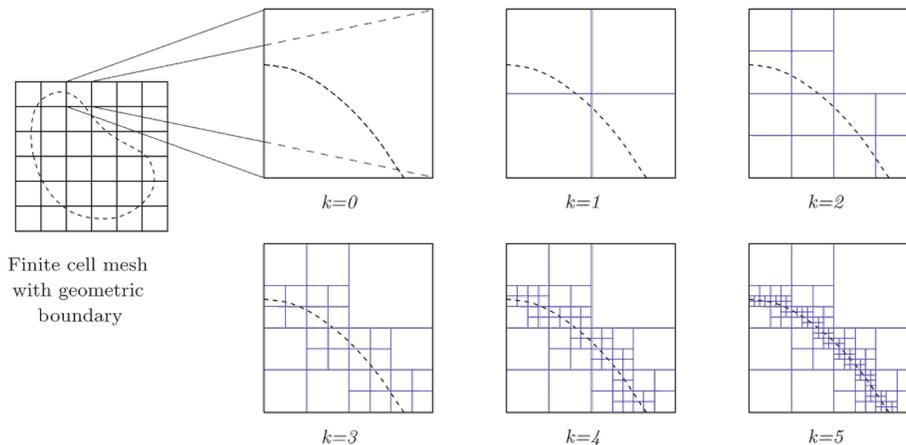


Figure 3: Adaptive integration of the cut elements

3 Multi-level *hp*-refinement

We combine the finite cell method with the multi-level *hp*-method³ to achieve high-order convergence rates even for singular problems. At the same time we add the possibility to discretize the computational domain around the laser with element sizes much smaller than for regions further away. The multi-level *hp*-method is a recently proposed variant of the *hp* finite element method. It eases the implementational effort usually being associated with hanging nodes, edges and faces.

The idea of the multi-level *hp*-method is to use hierarchical, high-order overlay meshes for a local enrichment of the discretization (see figure 4). These high-order enrichments need to be both linear independent and maintain compatibility at hanging entities. This is easily achieved, also in three dimensions by the following two simple rules: (a) Linear independence is ensured by deactivating all topological components that have active sub-components. (b)

³N. Zander et al. “Multi-level *hp*-adaptivity: high-order mesh adaptivity without the difficulties of constraining hanging nodes”. In: *Computational Mechanics* 55.3 (2015), pp. 499–517.

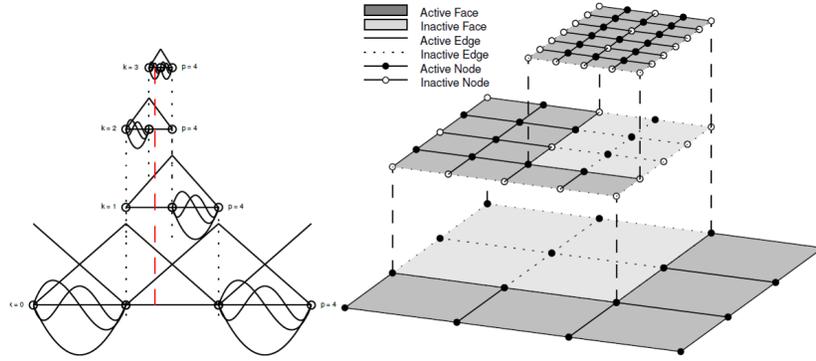


Figure 4: Conceptual idea of multi-level hp -refinement

Compatibility is ensured by homogeneous Dirichlet boundary conditions on the overlay meshes.

Recent investigations show that the multi-level hp -method yields the same excellent approximation capabilities as conventional hp -techniques.

4 Thermo-mechanical model of the SLM process

The principle of the SLM process is shown in figure 5a. It constitutes a multi-physics problem, where the highest temperature gradients as well as the phase changes occur in close vicinity of the moving laser beam. The phase changes

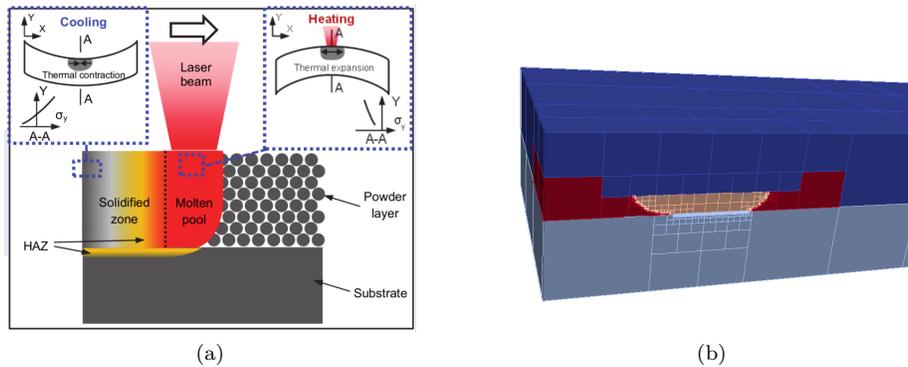


Figure 5: Selective laser melting process principle (a)⁴ and multi-level grid (b)

⁴C.H. Fu and Y.B. Guo. “Three-Dimensional Temperature Gradient Mechanism in Selective Laser Melting of Ti-6Al-4V”. in: *J. Manuf. Sci. Eng.* 136 (2014), pp. 061004–7

between powder, liquid and solid states are simulated by the model introduced by Celentano et. al.⁵ which uses the discretized weak form of equation (3).

$$\rho c \frac{\partial T}{\partial t} + \rho L \frac{\partial f_{pc}}{\partial t} - \nabla \cdot (k \nabla T) = Q \quad (3)$$

However, the effects of heat diffusion as well as the thermo-mechanical process of residual stress evolution need to be taken into account on all scales. For this purpose, a dynamically adaptable data container (multi-level grid) is used to keep track of the solidified domain Ω_{solid} locally and globally.

The container represents a dynamic octree. It stores the current material state of a voxel at all points in time. This additional tracking of the material state is needed since below the melting temperature the material can be either powder or solid and this in turn depends on whether solidification has already occurred previously.

Due to the changes in the temperature during the process, some regions in the domain expand, while some others contract (see figure 5a). This generates residual stresses in the part. They are computed by the quasi-static mechanical model, given in equation (4).

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{f} - \mathbf{f}^{th} = \mathbf{0} \quad (4)$$

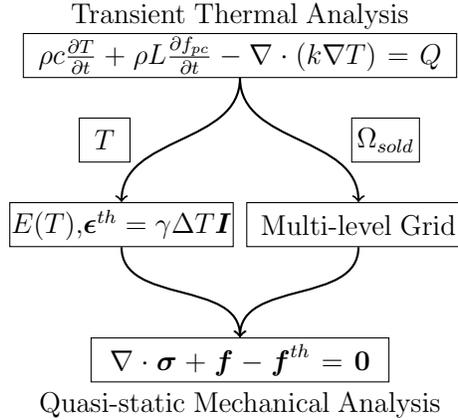


Figure 6: Coupling of thermal and mechanical problems

It is assumed that displacements are small and do not produce heat, so that only a one-directional coupling has to be taken into account, i.e. only the displacement field is affected by the changes in the temperature field. As shown in figure 6, we take a staggered approach for the solution of the thermo-mechanically coupled problem. For each time step, firstly the thermal problem

⁵D. Celentano, E. Oñate, and S. Oller. “A temperature-based formulation for finite element analysis of generalized phase-change problems”. In: *Int. J. Numer. Meth. Engng.* 37 (1994), pp. 3441–3465.

is solved to obtain the temperature distribution and the solidified domain Ω_{solid} . The resulting temperature as well as the state dependent (and temperature-dependent) material coefficients are then converted to equivalent strains used as a load to solve the mechanical problem before the next time step is computed.

5 Results

This section presents some preliminary results of the approach presented above. The problem setup, including the laser path and the dimensions of the domain is illustrated in figure 7. The material domain consists of Ti-6Al-4V alloy, being widely used in both aerospace and biomedical applications of SLM. The laser is modeled as a surface heat flux with a Gaussian distribution.

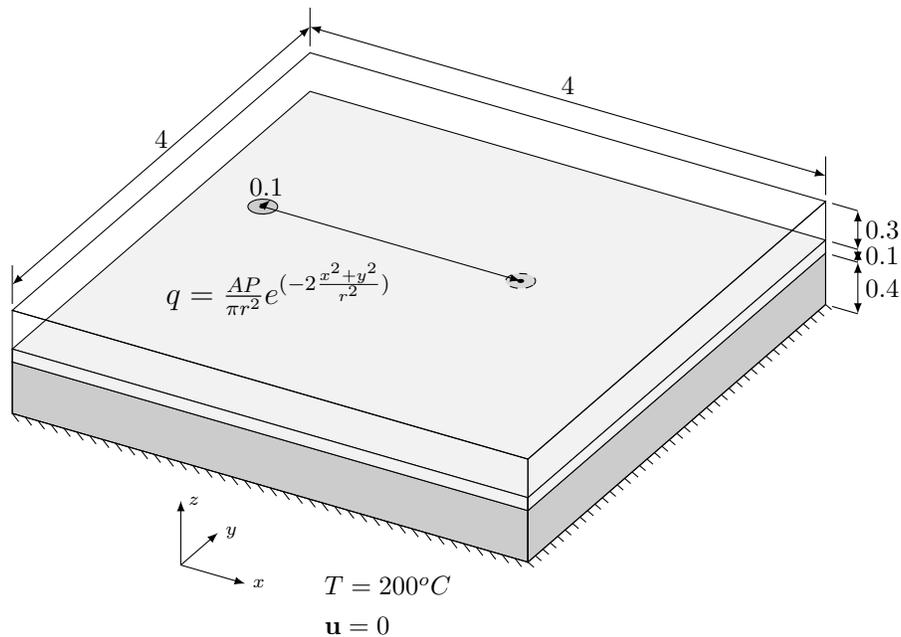


Figure 7: Simulation Setup

The results at the end of the simulation are depicted in figure 8. The temperature distribution over the entire domain is shown in figure 8a, while the distribution of the von Mises stress is presented in figure 8b only on the solidified domain. Moreover, the solidified part is warped with the displacement field to reveal the expansion and contraction of the part. It should also be noted that the front of the solidified material passes through the interior of finite cells, taking advantages of the underlying embedded domain approach.

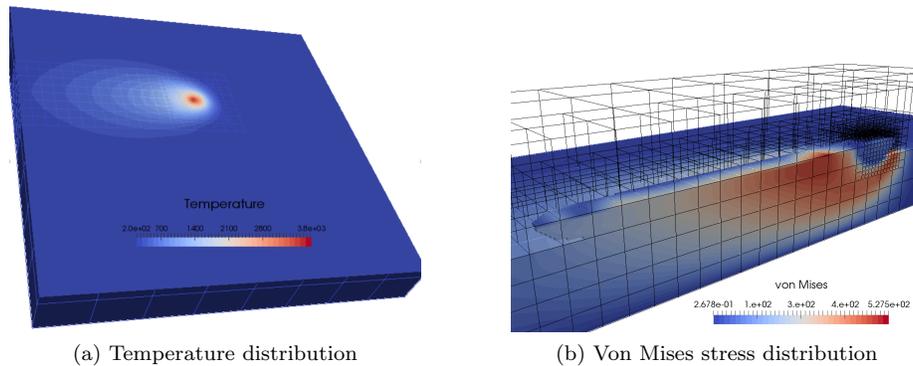


Figure 8: Results of the SLM simulation

6 Conclusions

We presented a geometrically flexible concept to simulate SLM processes in the framework of the Finite Cell Method, an embedded domain method of high order accuracy. It is geometrically flexible since the resulting geometry does not need to be resolved by the boundaries of the discretization of the field variables.

Moreover, the multi-scale nature of the SLM process motivated the development of a strictly hierarchical treatment of the state variables, the material coefficients and the field variables. To this end we enriched the Finite Cell Method with the multi-level *hp*-method and added a hierarchic, spatial treatment of the state variables using octrees.

We find our approach to be very promising in terms of computational efficiency and accuracy. However, it will surely not be sufficient to treat the vast scales involved in the prediction of mechanical properties of complete products produced by SLM. Future work at the Chair for Computation in Engineering will, thus, include methods to reduce the computational complexity by identifying relevant components of the simulation and using reduced order models to further increase the computational efficiency of the approach.

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