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Investigating traffic flow in the presence of hindrances by cellular automata *

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Abstract

Starting from a cellular automaton model (CA) for general conditions on a freeway we come up with a formulation of an automaton to include the case of hindrances on a road. Investigation of our model results in a phase diagram introducing a new phase between laminar and jammed traffic. This phase is characterized by the spatial coexistence of behaviour known from the original model.

1. Introduction

The growing number of cars involved in street traffic has lead to the investigation of traffic flow on freeways as early as in the early fifties. First treatment was based on models involving huge sets of difference or differential equations. Traditionally it was distinguished between microscopic and macroscopic models. Macroscopic models were usually based upon a continuum approach (see Prigogine [1], Lighthill [2] and Daganzo [3]). Because of the very high computational effort, microscopic models were not considered to be of high practical importance until recently computer capacity started to allow for the simulation of huge particle systems. In Germany this concept was pursued mainly by the 'Karlsruher Institut für Verkehrswesen'. The models developed there were all based on Wiedemann's 'Car-Following-Theory' [4] and empirical investigations done by Leutzbach [5]. As an alternative to those models CA (cellular automata) models for traffic flow were introduced. In [6] Nagel and Schreckenberg showed that they were able to reproduce the characteristic 'start-and-stop' waves found for freeway traffic under

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regular conditions by a probabilistic seven state CA. This model is further investigated in [7], where also a comprehensive bibliography on traffic simulation can be found. For general references on integer value based probabilistic CA rules see [8,9]. In Section 2 of the present paper a short review of this model is given. Section 3 then introduces our extension of the model to simulate hindrances on the road and gives results of our investigations. In Section 4 their possible relevance for real traffic is discussed. Problems concerning the model's validation are encountered as well.

2. Basic model

The model as proposed by Nagel and Schreckenberg (in the following referred to as the NaSc model) distinguishes seven possible states for each cell of the automaton – state 0 referring to an unoccupied cell whereas states 1 to 6 correspond to cells occupied by cars of velocity 0 to 5 respectively. The cells are arranged on a line modelling traffic flow in one direction only. The basic concept of cellular automata is to iterate discrete states of a cell system in discrete timesteps with each new cell state depending only on its own value and the one of its neighbors a timestep before. Nagel and Schreckenberg implement this concept by the following paralleled update rules:

(i) **Acceleration:** If the velocity of a vehicle is lower than v_{max} and if there is enough space ahead ($v \leq gap - 1$), then the speed is increased by one:
if ($v \leq gap - 1$) **then** $v := min[v_{max}, v + 1]$;
(ii) **Slowing down (due to other cars):** If the next car ahead is too close ($v \geq gap + 1$), speed is reduced to gap:
if ($v \geq gap + 1$) **then** $v := gap$;
(iii) **Randomization:** With probability p, the velocity of each vehicle (if greater than zero) is decreased by one:
with probability p : $v := max[v - 1, 0]$;
(iv) **Car motion:** Each car is advanced v sites;
with gap denoting the number of empty sites in front of the vehicle and v_{max} usually set to five.

The most important results of this model can be summarized as follows:
When plotted in a space–time diagram, the CA evolution displays the same 'start-and-stop' waves as known from Aerial Freeway Photographies [10]. Fig. 1 shows such a plot from a cellular automaton corresponding to a density of $\rho = 0.2$. The density on a road is defined as follows: $\rho = N/LENGTH$, with $LENGTH$ denoting the length of the automaton and N equals the number of occupied sites on the discretized road. Space direction is horizontal, the time coordinate is downwards, vehicles move to the right. Each black pixel represents one vehicle. The model evolves from random initial conditions subject to a predefined global density. The figure shows the first 420 iterations for a system of size $LENGTH = 1110$. However, only a window of 550 cells is plotted.

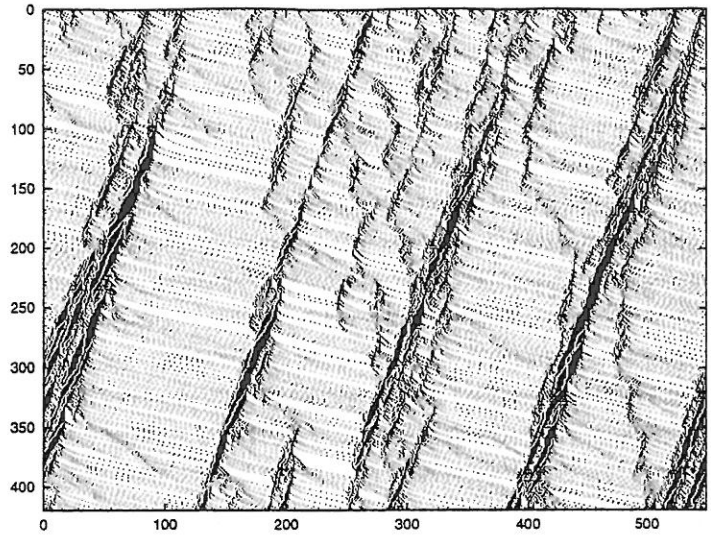

Fig. 1. Space–time plot for the NaSc model with ρ=0.2; the figure shows the first 420 iterations in a window of 550 cells. The system's length – denoted by *LENGTH* – is twice this size.

The evolution in the diagram depends on the density. Since we impose circular boundary conditions the density is constant during the system's temporal evolution. Whereas we find laminar traffic at low densities, there are congestion clusters (small jams) at higher densities. To characterize the dependency of the flow on density the quantity **flow** is introduced as follows[1]:

$$\langle q \rangle_{LENGTH} = \bar{v} \cdot \rho = \frac{1}{LENGTH} \sum_{i=1}^{N} v_i ,$$

where $\bar{v}$ is the mean velocity obtained by averaging over all cars of the system.

Plotting this quantity versus the density gives the so called fundamental diagram shown in Fig. 2. Each point corresponds to a fixed density. Simulations are carried out for 110 000 timesteps starting from a random initial distribution. The first 10^4 time steps are discarded to let the transient die out. Then, every 10^3 iterations one measurement step is inserted. The final data point is averaged over 100 such measurements. The curve shows 98 points for ρ varying from 0.01 to 0.99 in steps of 0.01. For the exact position of the curves' maximum, Nagel and Schreckenberg [6] give the value $(\rho_c, q_{max}) = (0.086 \pm 0.002; 0.318 \pm 0.001)$. Roughly speaking ρ_c separates the low density interval of laminar traffic flow – meaning that arising jams will eventually die out – from a 'jammed phase' characterized by the persistence of jams. At the critical point $\rho_c = 0.086$ itself the distribution of traffic jam lifetimes, $P(t)$, scales as $P(t) \sim t^{-3/2}$.[2]

[1] In some articles this quantity is also referred to as 'flux' or 'throughput'.

[2] Note that this picture is very rough. For detailed investigations concerning the criticality of the system see [11]. The actual scaling law proposed there is $P_{surv}(t, \Delta) \sim t^{-\delta} \cdot f(t \cdot \Delta^{\nu_t})$, where $\Delta = \rho - \rho_c$ and $P_{surv}(t) = \int_t^{\infty} dt' P(t')$. Simulation results for δ and ν_t are $\delta = 0.5 \pm 0.01$ and $\nu_t = 2 \pm 0.2$.

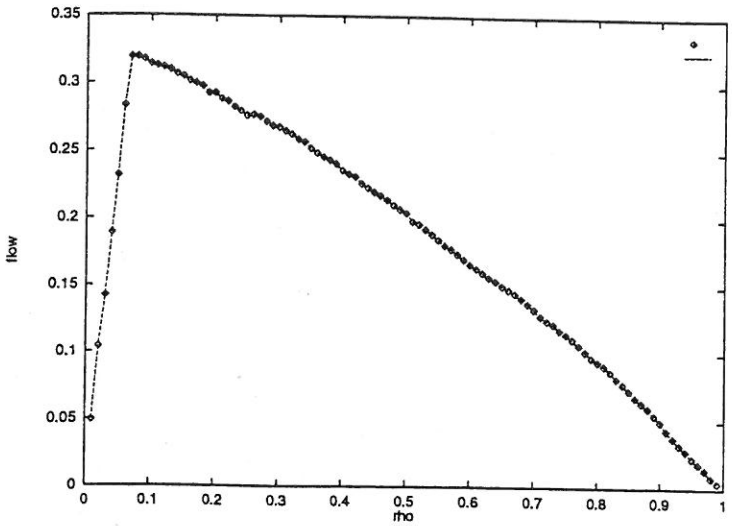

Fig. 2. Fundamental diagram for the NaSc model. For further explanation it is referred to Section 2.

3. Model with hindrances

The new feature of our model is a routine to simulate hindrances on a road. A similar problem was investigated in [12], where analytic expressions for average velocities and fluxes in the presence of 'impurity sites' were obtained for a two-speed model. Our approach generalizes the NaSc-model and imposes the following rule *before* all other update routines introduced in Section 2. It reads as follows:

$$\textbf{for } \left(i = \tfrac{1}{2}LENGTH \ to \ i = HIND - 1 + \tfrac{1}{2}LENGTH\right): \quad v := \tfrac{1}{2}v\,,$$

with $HIND$ giving the length of a hindrance positioned in the middle of the road. Integer arithmetics is used and $HIND$ is an integer value with a unit length equaling that of the automaton cells. The calibration used is that found in [6]: one cell corresponds to a road segment of length 7.5 m. $HIND = 1$ therefore denotes a disturbance on a street segment of length 7.5 m. As a first consequence of this additional rule, vehicles arriving at the hindrance with $v = v_{max}$ will reduce their speed over the first few cells to $v = 2$. A vehicle arriving in an already existing jam at the hindrance (i.e. with $v = 1$), will not be able to accelerate until the end of the disturbed segment.

To give an idea of the qualitative behaviour of this model Figs. 3 and 4 show the evolution of two CA's with $HIND = 2$ and $HIND = 3$ respectively ($\rho = 0.08$). Again the initial conditions are chosen randomly. The systems length is 4096, however only 1110 cells around the position 2048 (starting point of the hindrance) are displayed. This length was found to be twice the length necessary to avoid finite size effects leading to a slightly higher maximum flow for small systems. The time interval shown is $t = [1000, 1800]$.

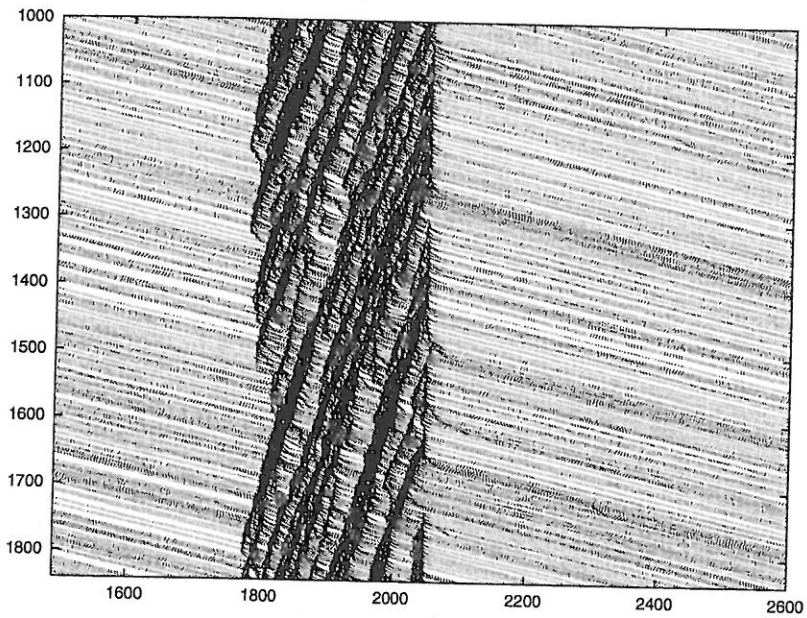

Fig. 3. Space–time plot for the model with a hindrance of length 2 at position 2048. 840 timesteps are shown beginning at $t = 1000$. In the x-direction 1110 cells around the hindrance are plotted.

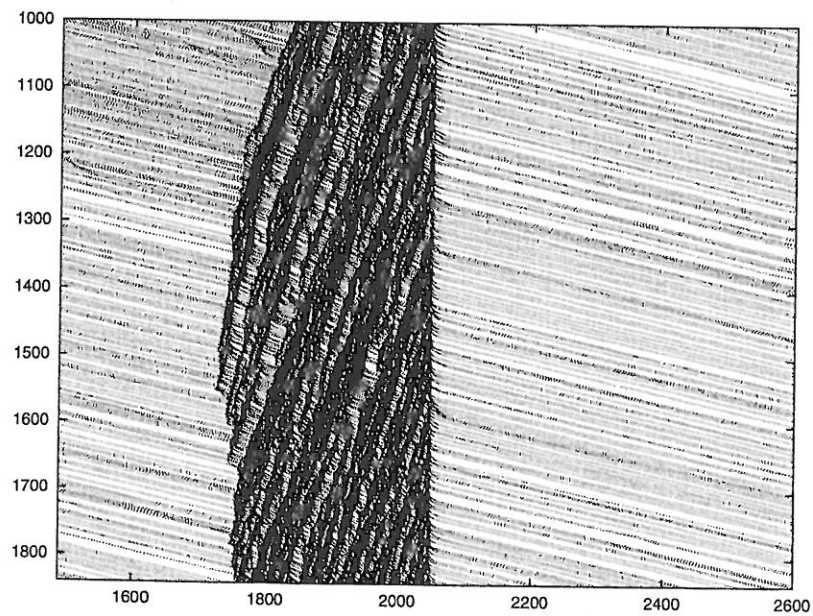

Fig. 4. Same plot as in Fig. 3, yet with a hindrance of length 3. Note the difference in density before the hindrance compared to Fig. 3.

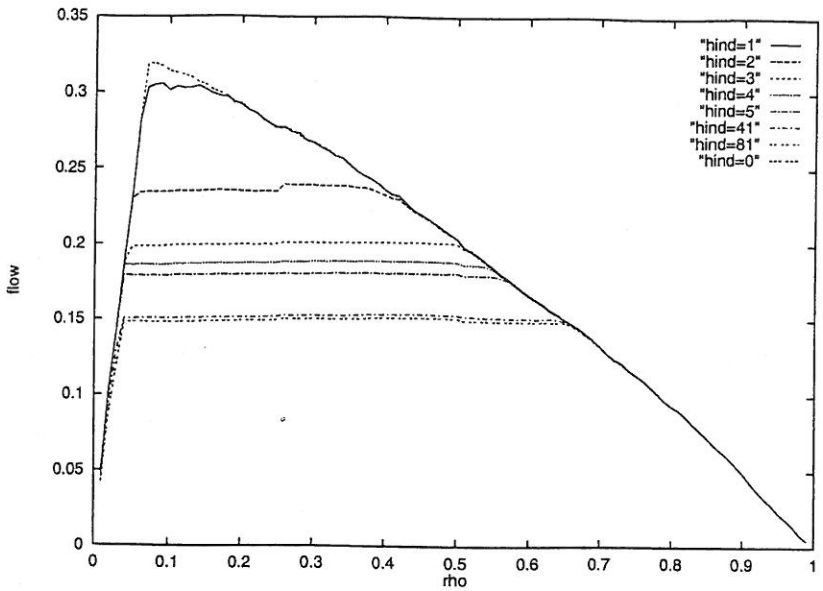

Fig. 5. Fundamental diagram for the model with hindrances of length 0 to 5, 41 and 81 respectively. Note that for $HIND = 0$ the model (and curve) is equivalent to the NaSc model.

In the middle of the system right before position $\frac{1}{2}LENGTH$ (and therefore right before the hindrance) one can find a 'jammed phase'[3]. This region is framed by a phase of free flow. The car density in both regions depends on the value for $HIND$. To get a better understanding of this dependency on $HIND$ we plot the fundamental diagrams for the models with $HIND = 0$ (thus corresponding exactly to the NaSc model), $HIND = 1$ to 5 and also $HIND = 41$ and 81 (see Fig. 5). The simulations for these diagrams were carried out as described in Section 2.

Investigation of this diagram now helps to understand the behaviour of our system in the following way:

For $HIND = 0$ only one 'critical' value of ρ leading to maximum throughput exists. For $HIND > 0$ however there is a whole density interval giving rise to q_{max}. This interval will be denoted by $[\rho_{c,min}(HIND);\ \rho_{c,max}(HIND)]$. The characteristic shape of the fundamental diagram is in very good agreement with observations, which were made by Latour [15], when he investigated two-lane traffic with a stopped car on the first lane. This correspondence could be expected, as his approach (also being based on the NaSc model), triggers a velocity reduction on the second lane.

The points $\rho_{c,min}(HIND)$ all belong to the monotonically increasing part of the fundamental diagram which can be fitted by a line from the origin to the maximum:

$$q(\rho) = 3.70\rho. \tag{1}$$

[3] The width of the jam (meaning its length in the x-direction of the diagram) will be referred to by $WIDTH$.

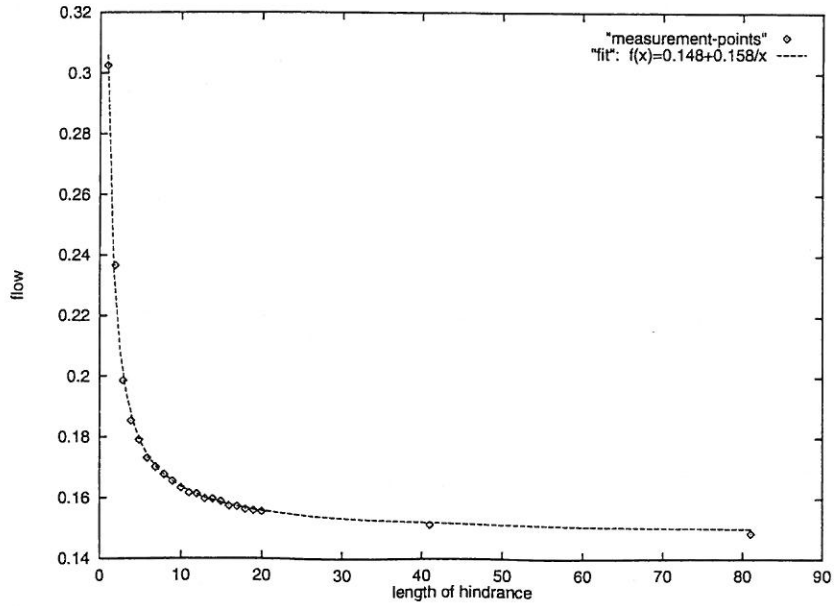

Fig. 6. Systems' maximum flow depending on the length of the hindrance.

To get an expression for the maximum flow depending on $HIND$ we plotted the corresponding diagram (see Fig. 6). Obtaining -1 as a slope in the double logarithmic plot, we fitted this curve by $1/HIND$, resulting in

$$q_{max}(HIND) = (0.148 \pm 0.001) + (0.158 \pm 0.04) \cdot \frac{1}{HIND} . \tag{2}$$

Setting (1) and (2) equal leads to the following $HIND$ dependency for $\rho_{c,\min}$:

$$\rho_{c,min}(HIND) = 0.04 + 0.04 \cdot \frac{1}{HIND} . \tag{3}$$

In the same way it is possible to fit the monotonously decreasing part of the fundamental diagram to obtain the expression

$$q(\rho) = 0.13 + 0.21[e^{-\rho} - \rho^2] . \tag{4}$$

It should be noted that we are presently not able to give a profound physical meaning to (4), it is just a (very precise) fit to the decreasing part of the curve.

Equalizing (4) and (2), we obtain[4]

$$HIND(\rho_{c,max}) = [1.3 \cdot (e^{-\rho_{c,max}} - \rho_{c,max}^2) - 0.105]^{-1} . \tag{5}$$

In Fig. 7 $HIND(\rho_{c,max})$ and $HIND(\rho_{c,min})$ (which can easily be obtained from (3) by solving for $HIND$) are plotted against the density. This gives rise to a phase diagram in which the 'free flow phase' characterised by a behaviour found for $\rho \in]0; 0.086]$

[4] The equation has to be given in this form since it can't be solved for $\rho_{c,max}$(HIND) explicitly.

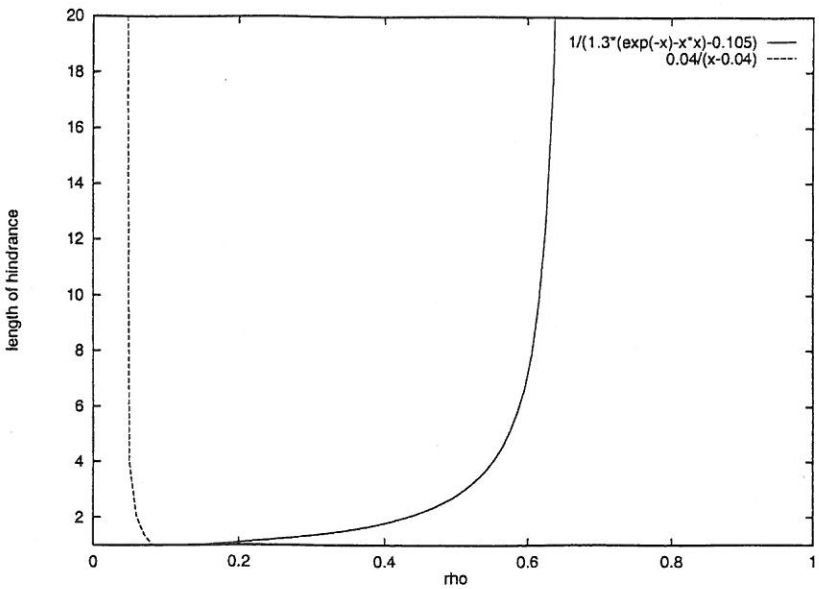

Fig. 7. Phase diagram: free flow left of dashed line; jammed phase right of solid line; jams induced by hindrance in transition phase between solid and dashed line.

in the NaSc model is positioned at the very left. At the very right we find a 'jammed phase' as Nagel and Schreckenberg do for $\rho \in [0.086; 1.0[$. The phase in between the two graphs shows a behaviour as displayed in Figs. 3 and 4. Quantitatively this can be described as follows:

- For the interval $[\frac{1}{2}LENGTH - WIDTH; \frac{1}{2}LENGTH]$ the system behaves like a CA of the NaSc model with density $\rho = \rho_{c,max}(HIND)$.
- Outside of that interval it displays 'free flow' corresponding to a NaSc-CA of $\rho = \rho_{c,min}(HIND)$.

The value for *WIDTH* can therefore be obtained from the following equation:

$$LENGTH \cdot \rho = \rho_{c,max} \cdot WIDTH + (LENGTH - WIDTH)\rho_{c,min} \tag{6}$$

$$\Updownarrow$$

$$WIDTH = LENGTH \frac{(\rho - \rho_{c,min})}{(\rho_{c,max} - \rho_{c,min})} . \tag{7}$$

Obviously *WIDTH* depends on *HIND* (since $\rho_{c,min}$ and $\rho_{c,max}$ depend on *HIND*) and on ρ. For $\rho = \rho_{c,max}$, *WIDTH* equals *LENGTH* meaning that the whole system behaves like a NaSc-CA for $\rho_{c,max}$. This corresponds to the fact that for this value of ρ up to $\rho = 1$ the fundamental diagrams for the NaSc model and the one with $HIND \neq 0$ are identical.

To sum up our numerical results it can be stated that even though there is a 'new' phase in the phase diagram of our model its local behaviour can be completely understood in

terms of the NaSc model since it can be separated into two regions each corresponding to a NaSc-CA for a different value of ρ. Thus also the scaling behaviour of the system can be referred back to the NaSc model.

4. Relevance for real traffic

The NaSc model was validated by data material from real traffic measurements as described in Section 2. Validating our results is a big problem due to the lack of empirical data for measured throughputs of road segments with hindrances. In that case only the mean velocity and its variances have been investigated so far [13]. Ref. [14] complains about this point and strongly advertises to begin those measurements. We want to stress the need for such investigations.

Nevertheless we want to point out that the qualitative behaviour of our system is plausible: In the outflow of a jammed region traffic actually does evolve to its free flow limit as can be observed in everyday traffic. The reason for this is the low density of that region. Just as well it is plausible that the maximum throughput through a hindrance sinks with its length.

The question that remains open is whether the critical values we obtained for systems with hindrances can be directly related to real traffic. If so our model could be a guidance for traffic planners dealing with routing and variable driving instructions. It predicts the critical value $\rho_{c,min}(HIND)$ which in case of existing hindrances has to guide shut down of a road rather than the critical density ρ_c obtained for undisturbed traffic. Further investigations of the model could be motivated by the idea of a dynamic jam warning system controlled by on-line density measurements.

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