SOME ASPECTS OF COUPLING STRUCTURAL MODELS
AND P-VERSION FINITE ELEMENT METHODS

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Abstract. This paper addresses some questions arising from an integration of coupling structural models and p-version finite element methods. After a brief introduction to the p-version for Reissner-Mindlin plate problems we will consider the modelling of loads or elastic foundations acting only on parts of elements. Furthermore we will address the question of a posteriori control of the accuracy of the approximation being important for reliable computations. The last part of the paper compares the computational efficiency of the p-version to low order approximations. Finally, it will be motivated why the use of the p-version is expected to be superior in parallel efficiency compared to standard h-version codes.
1 INTRODUCTION

The $p$-version of the finite element method has been investigated very intensively during the past 15 years and it has turned out to be superior to the classical $h$-version in a significant number of fields of practical importance. It has especially offered advantage when the finite element model is to be coupled to a structural model being defined in a CAD-environment. We will discuss preliminary results of a research project, where a $p$-version finite element code is implemented as one of several tasks in a civil engineering CAD-environment.

There are three crucial questions to be addressed when a finite element computation has to be embedded into a CAD frontend. The first is the problem of geometric flexibility. Although it is usually easy to modify the contour of a structure or the position of structural elements like columns supporting a plate concerning geometry aspects, it is still a time-consuming task to generate a finite element mesh for every modification of a structure. We will show that the $p$-version offers very attractive features to increase the flexibility of the finite element model. Significant modifications of the structure are possible without remeshing using strongly distorted elements, element shapes being described by much more complex functions as in the usual approach and, for plate problems, a modelling of local conditions independent of the finite element mesh is possible.

The second question concerns a posteriori control of the accuracy of the approximation, becoming even more important if the finite element computation results are used by an engineering designer and not a specialist in numerical simulation. We will present numerical results of a generalization of the residual type error estimation going back to the fundamental work of BABUŠKA and co-workers and being now common procedure for low order elements.

The third problem to be addressed will be computational efficiency, as an embedding for FEA in CAD is only of practical interest, if wallclock time for non-trivial computations is short. This question will be addressed in our paper discussing possibilities of parallelization of a $p$-version code.

2 THE $P$-VERSION

While in the standard $h$-version of the finite element method the mesh is refined to achieve convergence, the polynomial degree of the shape functions remains unchanged. Usually low order approximation of degree $p=1$ or $p=2$ is chosen. The $p$-version leaves the mesh unchanged and increases the polynomial degree of the shape functions locally or globally. In most implementations a hierarchical set of shape functions is applied, providing a simple and consistent facility of implementation in 1-, 2- or 3-dimensional analysis. Oscillations in the approximate solution, which could be expected when working with high order shape functions can be avoided by using properly designed meshes. Guidelines to construct these meshes a priori can often be given much easier for the $p$-version than for the $h$-version\footnote{For linear elliptic problems it was also proven mathematically that a sequence of meshes...}.
can be constructed so that the approximation error only depends on the polynomial degree \( p \) and not on the order of singularities in the exact solution.

Our \( p \)-version implementation uses hierarchical basis functions on quadrilaterals with an ansatz space suggested by Szabó and Babuška\(^2\). The definition of the polynomial spaces applied in this report are given in the following.

1. The \textit{reduced ansatz space} is the space of polynomials on \( \Omega^p = [(-1, 1) \times (-1, 1)] \) which is spanned by the set of all monomials
   \begin{itemize}
   \item \( \xi^i \eta^j \), \( i, j = 0, 1, \ldots, p; i + j = 0, 1, \ldots, p \)
   \item \( \xi \eta \) for \( p = 1 \)
   \item \( \xi^p \eta, \xi \eta^p \) for \( p \geq 2 \)
   \end{itemize}

2. The \textit{full ansatz space} is the space of polynomials on \( \Omega^p = [(-1, 1) \times (-1, 1)] \) which is spanned by the set of all monomials
   \begin{itemize}
   \item \( \xi^i \eta^j \), \( i, j = 0, 1, \ldots, p \)
   \end{itemize}

Another main difference between \( h \)- and \( p \)-version finite element methods lies in mapping requirements. Because in the \( p \)-version the element size is not reduced as the degrees of freedom are increased the description of the geometry must be independent of the number of elements. This results in the necessity to construct elements with an exact representation of the boundary. The isoparametric mapping, used in standard finite element formulations, can be seen as a special case of mapping using the \textit{blending function method}\(^2,\(^3\). Following these ideas element boundaries can be implemented as (almost) arbitrarily curved edges.

In addition to the \textit{geometric flexibility} of higher order elements there are several reasons for the attractiveness of higher order elements like their \textit{high accuracy} and their \textit{robustness}. High accuracy is due to an exponential rate of convergence in the case of an analytical exact solution which can even be obtained for problems with singularities when an increase of the polynomial order is combined with local mesh refinement in an \( hp \)-version. The robustness of the \( p \)-version allows the use of strongly distorted elements and prevents from Poisson ratio locking in cases of nearly incompressible materials and from shear locking in thin plate situations when the Reissner-Mindlin theory is used, see e.g. Holzer et.al.\(^4\).

### 3 THE REISSNER-MINDLIN PLATE PROBLEM

Consider a plate of thickness \( t \) bedded on an elastic foundation imposed to a load \( p_z(x, y) \) in \( z \)-direction, as shown in Figure 1. The middle surface \( \Omega \) with boundary \( \Gamma \) of the plate is assumed in the \( x-y \) plane. The mode of deformation of a point with coordinates \((x,y,z)\) in a Reissner-Mindlin plate is represented by

\[
\begin{align*}
  u &= \beta_x(x, y) \cdot z, \quad v = \beta_y(x, y) \cdot z, \quad w = w(x, y)
\end{align*}
\]  

\(1\)
where $u, v$ and $w$ are the displacement components in $x, y$ and $z$ directions respectively, $\beta_x$ and $\beta_y$ are the normal rotations in $x$-$z$ and $y$-$z$ planes. The sign convention for the stress resultants, namely the bending moments $M_x, M_y$, the twisting moments $M_{xy} = - M_{yx}$ and the shear forces $Q_x, Q_y$ are also shown in Figure 1. In order to formulate a finite element approximation for the given boundary value problem we consider the weak formulation:

Find $\mathbf{u} = \left[ \beta_x^{(u)}, \beta_y^{(u)}, w^{(u)} \right]^T \in E(\Omega)$ such that $\forall \mathbf{v} = \left[ \beta_x^{(v)}, \beta_y^{(v)}, w^{(v)} \right]^T \in \bar{E}(\Omega)$

$$
B(\mathbf{u}, \mathbf{v}) = \mathcal{F}(\mathbf{v})
$$

where the bilinear functional $B : E(\Omega) \times \bar{E}(\Omega) \to \mathbb{R}$ and the linear functional $\mathcal{F} : \bar{E}(\Omega) \to \mathbb{R}$ define the internal and external virtual work. The space of kinematically admissible displacement functions $E(\Omega) = \{ \mathbf{v}(x) \in (H^1(\Omega))^3 : \mathbf{v} = \mathbf{0} \text{ on } \Gamma_D \}$ represents the set of displacement functions which have finite energy on $\Omega$ and satisfy the prescribed kinematic boundary conditions whereas $\bar{E}(\Omega) = \{ \mathbf{v}(x) \in (H^1(\Omega))^3 : \mathbf{v} = \mathbf{0} \text{ on } \Gamma_D \}$ defines the space of displacement functions which have finite energy and vanish on $\Gamma_D$. The bilinear form

$$
B(\mathbf{u}, \mathbf{v}) := \frac{\nu^3}{12} \int_\Omega \epsilon_B^{(v)^T} D_B \epsilon_B^{(u)} d\Omega + \int_\Omega \epsilon_S^{(v)^T} D_S \epsilon_S^{(u)} d\Omega + \int_\Omega \mathbf{u}^{(v)^T} C \mathbf{u}^{(u)} d\Omega
$$

is defined as the sum of a bending term, a shear term and a part due to an elastic foundation. The strains are defined as

$$
\epsilon = [\epsilon_B^T, \epsilon_S^T]^T = \left[ \frac{\partial \beta_x}{\partial x}, \frac{\partial \beta_y}{\partial y}, \left( \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \right) z, \beta_x + \frac{\partial w}{\partial x}, \beta_y + \frac{\partial w}{\partial y} \right]^T
$$

where $\epsilon_B z$ and $\epsilon_S$ denote strain components caused by bending and shear, respectively. The positive definite matrices

$$
D_B = \frac{E}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}, \quad D_S = \frac{E}{2(1+\nu)} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & 0 & 0 \\ 0 & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix}
$$
define the assumed linear elastic behaviour of the Reissner-Mindlin plate and the foundation. \(E, \nu\) are Young’s modulus and Poisson’s ratio, respectively, and \(k\) is the warping coefficient accounting for non-uniform shear distributions which is often set to \(k = \frac{3}{8}\). The coefficients \(c_{11}, c_{22}\) and \(c_{33}\) define a linear elastic foundation. The load functional is given by

\[
\mathcal{F}(\nu) := \int_{\Omega} p_z w^{(v)} d\Omega + t \int_{\Gamma} \left( M_n \beta_n^{(v)} + M_t \beta_t^{(v)} - Q w^{(v)} \right) d\Gamma. \tag{6}
\]

The relation between stress resultants and strains is given by the constitutive equations

\[
M = \begin{bmatrix} M_x, M_y, M_{xy} \end{bmatrix}^T = -\frac{t^4}{12} D_B \varepsilon_B, \tag{7}
\]

\[
Q = \begin{bmatrix} Q_x, Q_y \end{bmatrix}^T = -t D_S \varepsilon_S.
\]

4 MODELLING OF COLUMNS AND LOADS USING THE P-VERSION

A finite element analysis is only one of many tasks in an iterative engineering design process. It is therefore necessary to support quick and easy modifications of the finite element model. The p-version offers an attractive feature to increase the flexibility of the finite element model. Significant modifications of the structure are possible without remeshing.

To be more precise, we consider the Reissner-Mindlin model problem defined in section 3. Using the h-version each (re)definition of loads acting only on parts of the plate or columns supporting the plate requires remeshing in order to resolve the geometry of the loaded or supported areas. Furthermore it is necessary to increase the number of elements in the area of loaded or supported parts of the plate to provide an approximation with an acceptable error. In the following subsection we will present a method feasible only for the p-approach based on a composed integration scheme and supporting quick and easy modifications of the finite element model.

4.1 Modelling of columns

Figure 2 shows an example of how a column can be modelled independent of its position with respect to the finite element mesh. The plate \(\Omega = \bigcup \{\Omega^e, e \in I_e\}\) consisting of 16 p-elements is supported on a subdomain \(\Omega_C\) bounded by a piecewise linear boundary \(\Gamma_C\). The domain of the column \(\Omega_C\) is then subdivided \(\Omega_C = \bigcup \{\Omega_k, k \in I_k\}\) automatically using standard mesh generating techniques. It has only to be taken care that each subelement \(\Omega_k\) can be allocated uniquely to a p-element \(\Omega^e\). The subdivision of \(\Omega_C\) can now be used to perform a composed integration to compute the additional stiffness due to the elastic foundation. If we consider e.g. the calculation of the stiffness matrix corresponding to element 10 we have

\[
K^{10} = \int_{\Omega^{10}} \frac{t^3}{12} B^T_B D_B B^T_B + t B^T_S D_S B^T_S d\Omega^{10} + t \sum_{k=1}^{7} \int_{\Omega_k} N^{10T} C N^{10} d\Omega_k. \tag{8}
\]
It is important to notice that the subdivision of the domain $\Omega_C$ serves for integrating the term $N^{10T}CN^{10}$, where $N^{10}$ is the matrix of shape functions defined for the p-element e=10, and not for the definition of additional shape functions.

4.2 Modelling of forces

Analogously we apply the composed integration scheme to model forces defined only on parts of the domain $\Omega$. A force acting on $\Omega_C$ in the example of section 4.1 would result in an element force vector:

$$f^{10} = t \sum_{k=1}^{7} \int_{\Omega_k} N^{10T} p_z d\Omega_k$$  \hspace{1cm} (9)

4.3 A numerical example

To demonstrate the accuracy of the p-version in connection with the composed integration scheme we consider the plate shown in Figure 3. Along all bold lines of the plate structure soft simple support ($w = 0$) is defined, at the remaining boundary part clamped boundary conditions ($\beta_x = \beta_y = w = 0$) are chosen. Furthermore the plate is supported by four columns each of the size 0.4 m × 0.8 m. The shaded areas mark the regions in which the plate is loaded by a uniform pressure of 35.0 kN/m$^2$. The columns are defined as an elastic foundation with $c_{11} = c_{22} = 0$, $c_{33} = 12400000$ kN/mm$^2$.

Figure 4 shows two meshes: Mesh (a) consists of 47 quadrilateral elements. Each column is discretized by 4 elements. Mesh (a) will be used to produce a reference solution where the polynomial degree of the elements is chosen to be $(p_\beta, p_w)=(11,12)$\textsuperscript{1}, based on the reduced ansatz space. Mesh (b) consists of only 11 quadrilateral elements. The polynomial degree of the elements is $(p_\beta, p_w)=(9,10)$, again based on the reduced ansatz

\textsuperscript{1}The polynomial degree can be chosen separately for the rotations $\beta_x, \beta_y$ and for the deflection $w$. 

Figure 2: Plate with an elastic foundation
space. The columns will be modelled as an elastic foundation applying the composed integration scheme described in the previous section. Each column is therefore subdivided into 16 quadrilateral subdomains to calculate the additional stiffness due to the elastic foundation.

Figure 3 and Figure 6 show a comparison of the reference solution and the approximation obtained by the composed integration scheme. Bending moments $M_x$ along the section lines A-A and B-B are plotted. It can be seen that the error of the bending moment $M_x$ along the section line A-A is negligible. Even the bending moment $M_x$ along the section line B-B shows only small deviations from the reference solution. The small errors of the approximation are due to the rich ansatz space provided by the p-elements. It should be noted again that a change of size or position of the columns can now be handled easily with the method presented, without a remeshing being necessary in this situation when performing an h-version computation.
Figure 4: Mesh (a), mesh (b): Two different discretizations for a plate with four columns

Figure 5: Bending moment $M_x$ along section line A-A
5 A POSTERIORI ERROR ESTIMATION

For an a posteriori error estimation we consider the concept first developed by BABUŠKA and MILLER for the problem of plane elasticity. This concept was extended to h-version Reissner-Mindlin problems. Following the heuristic generalization for high-order shape functions, given by KELLY et al. we choose the following error indication

\[ \| e \|_{E(\Omega)}^2 \approx \eta^2 = \sum_{i=1}^{n_{el}} \lambda_i^2 \]  

(10)

\[ \lambda_i^2 = \frac{h_i^2}{24p^2 E_{B,i}} \int_{\tilde{h}_i} (r_{1,i}^2 + r_{2,i}^2) \, d\Omega_i + \frac{h_i^2}{24p^2 E_{S,i}} \int_{\Omega_i} r_{3,i}^2 \, d\Omega_i + \frac{h_i}{24p E_{B,i}} \int_{\Gamma_i} \left( J_{1,i}^2 + J_{2,i}^2 \right) \, d\Gamma_i + \frac{h_i}{24p E_{S,i}} \int_{\Gamma_i} J_{3,i}^2 \, d\Gamma_i \]  

(11)

where \( p \) is the polynomial degree of the finite element approximation, \( E_{B,i}, E_{S,i} \) are the largest eigenvalues of the elasticity matrices (5), \( h_i \) is the diameter of element \( i \), \( J_{1,i}, J_{2,i}, J_{3,i} \) and \( J_{3,i} \) are the jumps of stress resultants along the element boundary \( \Gamma_i \) and \( r_{1,i}, r_{2,i} \) and \( r_{3,i} \) are the residuals of the finite element approximation on \( \Omega_i \). To compute the jumps of the stress resultants the stresses are transformed to the normal and tangential direction of
the element edges. If the concerned edge of the element $i$ is adjacent to another element, the jumps are defined as the differences of the stress resultants at the two joint element edges. If the edge is part of the Neumann boundary $\Gamma_N$, the jumps are defined as the difference of the stress resultants at the element edge and the constrained stress. In all other cases the jumps of the stress resultants are set to zero. Equation (10) will be used to estimate the error in energy norm for the p-elements.

5.1 Numerical examples

We will present two numerical examples to show the performance of the residual type error estimator (10). For both examples a reference solution is computed with the p-version on a fine mesh with up to 75000 degrees of freedom. The energy of the 'exact' solution is computed by Richardson extrapolation from the sequence of p-solutions.

5.1.1 Quadratic plate

As a first example we consider a quadratic plate with clamped boundary conditions ($\beta_x = \beta_y = w = 0$). The plate is loaded by a uniform area load. Figure 7 shows the finite element mesh refined to resolve the boundary layer of the Reissner-Mindlin solution. The diameter of the refinement layer equals the thickness of the plate.

![Finite element mesh for a quadratic plate](image)

A finite element approximation is computed with polynomial degrees varying from $p=1$ up to $p=8$ using the full ansatz space. Figure 8 shows a comparison between the estimated and the 'exact' error. It can be seen that the estimation performs quite well. The efficiency index $\theta = \frac{\|\tilde{e}\|_{L^2}}{\|e\|_{L^2}}$ being a measure for the quality of the error estimation lies between 0.98 and 1.5.

5.1.2 L-shaped plate

The second example represents an L-shaped plate with a uniform area load. The plate is clamped ($\beta_x = \beta_y = w = 0$) at one of the long edges. A finite element mesh, Figure 9,
Figure 8: Relative error in energy norm

The mesh is constructed with four layers of refinement towards the singular vertex at the reentrant corner. The progression factor is chosen to be 0.15 as recommended by Szabo1.

Figure 9: Finite element mesh for a L-shaped plate

Again, the full ansatz space was chosen. The polynomial degree varies between p=1 and p=15. Figure 10 shows a comparison between the estimated and the 'exact' error with an efficiency index \( \theta \) between 0.99 and 1.2.

Although the performance of the residual type error estimator performs very well for the chosen discretizations it has to be mentioned that for the p-extension on a mesh where singularities are not suffiently refined, the accuracy of the error estimator deteriorates.
significantly. A possible explanation for this behaviour may be the error pollution effect of the singularity which is only isolated by the locally refined meshes shown in our examples.

6 INVESTIGATION OF THE P-VERSION WITH RESPECT TO COMPUTATIONAL EFFICIENCY

Typically, the FEM computation can be divided into several subprocesses requiring different amounts of time:

- definition of the system (preprocessing): $T_{pre}$
- mesh generation: $T_{mesh}$
- computation of stiffness matrices and load vectors: $T_{st}$
- solution of the generated equation system: $T_{eq}$
- post-processing: $T_{post}$

For the h-version, the biggest amount of CPU time is usually needed for the solution of the equation system. For the p-version the situation is different: For $p \geq 4$ the computation of the element stiffness matrices usually takes more time than solving the equation system and special techniques are available to shift the amount of time further.
from $T_{eq}$ to $T_{st}$ and $T_{post}$. Considering e.g. a finite element for the Reissner-Mindlin plate theory based on hierarchical shape functions and full ansatz space with degree $p=8$ the size of the stiffness matrix is $n = 243$. The amount of degrees of freedom corresponding to the bubble modes is 147 being about 60% of the total number of degrees of freedom on element level. As the bubble degrees of freedom are purely local to the element, they can be condensed using a modified Cholesky decomposition for the element stiffness matrices. This results in further increase of $T_{st}$ and drastic decrease of $T_{eq}$ because the condition number of the global stiffness matrix is strongly reduced. Several authors have investigated these observations in detail interpreting the bubble mode condensation as a preconditioning procedure.

To confirm this observation, execution times and iteration counts for $p$-extensions with full ansatz space on uniform $3 \times 3$- and $12 \times 12$-meshes for the rhombic plate problem, Figure 11, are compared to an h-version, using a refined mesh with more than 9000 MITC4-elements. For the theory of the MITC4-element we refer to Bathe/Dvorkin. This low-order computation was performed using a classical conjugate gradient solver with incomplete Cholesky preconditioning. For the $p$-extension, the conjugate gradient solver was preconditioned by diagonal scaling, after elimination of all element bubble modes. All computations were performed on a HP735/125-workstation. Table 1 shows the number of degrees of freedom, the number of iterations, time needed for equation solution and total execution time. Elimination of bubble modes, e.g. the major step of preconditioning the $p$-version is not included in equation solution time, as it is performed completely on element level during pre- and post-processing. Note that the number of iterations in the $p$-extension only grows very slowly with the $p$-degree, and that the relative amount of work is shifted drastically from equation solution to the element level with increasing $p$.

As the most CPU intensive part for higher $p$-levels is the computation of the element stiffness matrices it should be noted that these computations can be naturally parallelized as they do hardly require communication. A similar argument holds for post-processing computations which take relatively more time for the $p$- than for the h-version.
<table>
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<th>Number of elements</th>
<th>Number of dof</th>
<th>Comp. on element level (sec)</th>
<th>CG solution (sec)</th>
<th>iterations</th>
<th>Total time (sec)</th>
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</table>

Table 1: Computational effort for p-extensions compared to MITC4-element

6.1 A practical example

Finally we show a 'real life' testcase to verify above observations for a non-academic problem. We consider therefore a slab of a parking garage supported by 30 columns and loaded by own weight, see Figure 12. The columns are defined as an elastic foundation with $c_{11} = c_{22} = 0, c_{33} = 12400000 \text{MN}^{m}{m}$. Furthermore parts of the slab are supported by walls, marked as dotted lines. The plate is discretized by mesh (b), Figure 13, consisting of 139 elements with $p=8$ and full ansatz space. The additional stiffness due to the elastic foundation is computed by the method presented in section 4.1. Again, the conjugate gradient solver was preconditioned by diagonal scaling, after elimination of all element bubble modes. Table 2 shows the time of computation performed on a HP C100-workstation. A reference solution is obtained by mesh (a) consisting of 300 quadrilateral elements with $(p_{n}, p_{w})= (11,12)$ and reduced ansatz space. Each column is discretized by one p-element. The corresponding number of degrees of freedom is 54781. In Figure 14 and Figure 15 the results of the computations are compared. Deflection $w$ and bending moment $M_y$ are plotted along the coordinate $y$ of section line F-F. It can be seen that the results, obtained on the coarse mesh (b) are very well within the tolerance acceptable for engineering purposes.
Figure 12: 'Real life' plate example with 30 columns

Figure 13: Mesh (a), mesh (b): Two different discretizations for a plate with 30 columns
Figure 14: Deflection $w$ along section line F-F

Figure 15: Bending moment $M_y$ along section line F-F
7 CONCLUSIONS

The purpose of this article was to show the attractiveness of the p-version with respect to a coupling of a CAD frontend and finite element computations in civil engineering. As a model problem we chose the Reissner-Mindlin plate theory. Three crucial topics have been adressed to discuss the embedding of finite element computations in a CAD frontend. First the geometric flexibility of the p-version was demonstrated by the ease of modelling forces and elastic foundations acting only on parts of elements. It was shown that significant modifications of the structure are possible without remeshing. Another important topic is a posteriori control of the accuracy of finite element approximations. Two numerical examples demonstrated that reliable computations can be performed by monitoring the accuracy of finite element computations using a residual type error estimator. In the last part of the paper the computational efficiency of the p-version was compared to the h-version and it turned out, that our p-code is comparable in speed to h-version computations. Furthermore we motivated the use of the p-version for parallel computations and examined the influence of the condensation of bubble modes on the overall efficiency. We expect that for higher p-levels the p-version will be superior in parallel efficiency as compared to a classical h-version approach.

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REFERENCES


