Parallel Computing
Exercise Sheet 4: Static & Dynamic Networks
28.11.2017

1 Performance Evaluation: Torus vs. Hypercube

a) Given is a 3-dimensional torus with size $8 \times 8 \times 16$ (i.e. 1024 nodes).
   I. Calculate the bisection width of this network.
   II. Calculate the diameter of this network.
   III. Calculate the cost of this network.

b) Given is a 10-dimensional hypercube (i.e. 1024 nodes).
   I. Calculate the bisection width of this network.
   II. Calculate the diameter of this network.
   III. Calculate the cost of this network.

c) Dependent on bisection width, diameter, and cost of the two networks in a) and b),
decide on basis of a cost-benefit ratio which topology to choose for a 1024
processor machine. Discuss your decision!

2 Network Topologies

Shown are the first levels of a recursive network topology – a pyramid. Each node always
has four children on the next level and all those children nodes on the next level are
connected in form of a 2-dimensional mesh.

![Pyramid Network Example](image)

Fig 1: example of a pyramid network of height $H = 2$ with $N = 21$ nodes

a) Give a closed formula for calculating the total number of nodes $N$ of a general pyramid
network depending on $H$, i.e. the height of the network only.

b) Calculate cost (i.e. the amount of edges), diameter, and bisection width of a general
pyramid network depending on $N$ and $H$ only.
3  Contention-Free Routing

The Beneš multistage network is one possible dynamic topology for the interconnection of nodes in a cluster system. Key property of this topology is the existence of a contention-free routing for all permutations of inputs to outputs. Shown below is an example of a Beneš network with 8 inputs/outputs.

![Beneš network diagram]

a) Show the following non-blocking parallel communications for the above network:

- 0 → 3
- 1 → 5
- 2 → 4
- 3 → 7
- 4 → 6
- 5 → 0
- 6 → 1
- 7 → 2

b) Explain (or prove) why the Beneš network is non-blocking.

4  Rearrangeable vs. Strict-Sense Non-Blocking

To reduce the total number of necessary switching elements in case of crossbars for interconnecting $K$ inputs with $K$ outputs, CLOS networks consist of three stages (ingress, middle, and egress) of much smaller crossbars, i.e. with less than $K^2$ switching elements. Hence, the ingress stage contains $R$ crossbars with $N \times M$ inputs/outputs, the middle stage contains $M$ crossbars with $R \times R$ inputs/outputs, and the egress stage contains $R$ crossbars with $M \times N$ inputs/outputs. Depending on the relative values of $M$ and $N$, rearrangeable non-blocking ($M \geq N$) and strict-sense non-blocking ($M \geq 2N-1$) can be distinguished.

a) Draw a strict-sense non-blocking CLOS network with properties $R = N = 2$. Show the following parallel communications:

- 0 → 3
- 1 → 1
- 2 → 0
- 3 → 2

b) Draw a rearrangeable non-blocking CLOS network with properties $R = N = 3$. Show one situation in which rearrangement is necessary.