1 Synchronisation

A railway track between two cities contains a bridge over a canyon that can be accessed by a single train at any point in time only. Hence, this bridge is an exclusive usable resource and needs to be implemented as a critical section.

There exist two types of trains: R-trains that always drive from the left-hand to the right-hand side and L-trains that always drive from the right-hand to the left-hand side. Give a valid synchronisation (pseudo code) using as many semaphores as necessary, thus, no two trains can access the bridge at the same time and R-trains and L-trains access the bridge alternately, i.e. after an R-train always follows an L-train and vice versa. Also give a correct initialisation of all your semaphores!

2 Diffusion Model

Given is a 2-dimensional ILLIAC mesh with $4 \times 4$ nodes. The single nodes have an initial load situation, i.e. number of tasks still to be processed, as follows.

Compute the first two load balancing steps based on a diffusion model for $\alpha = 0.10$ using total-step iteration (i.e. for $w^{(k+1)}$ only values $w^{(0)}$ are used) as update scheme!
3 Data Distribution and Efficiency

Given is some iterative algorithm that processes 3-dimensional data stored in a 3-dimensional matrix of side length $N$. Processing one element $a_{ijk}$ of $A$ takes $t_{comp} = 2\text{ns}$ time. Exchanging one element between two processes (one direction!) takes $t_{comm} = 0.5 \cdot t_{comp}$ time. After one complete processing step all processes have to exchange data at their borders (treat all processes the same and do not distinguish between different numbers of neighbouring processes!).

Choose a row-wise, column-wise, or block decomposition of $A$ and sketch the according data distribution. For which sizes $N$ of $A$ (depending on the number of processes $p$ only) could a parallel efficiency of at least 80% according to your decomposition be achieved?

4 Network Topologies

A recursive 2-dimensional network topology is constructed as follows. Starting from a single square (level 1), each edge is cut into three equal parts where the middle part is replaced by a square (level 2) without any rotations half above and half below this edge. This approach will then be recursively repeated on level $K$ for all squares of level $K-1$ until the final level has been reached. The nodes of this network are placed on the corners of the squares on all levels only.

a) Draw the first three iterations of this network!

b) Calculate (in closed form)

   I. the total number of nodes and
   II. the diameter

   of such a network with $K$ levels depending on $K$ only!