A formulation for frictionless contact using a material model and high order finite elements

Background

Contact problems in solid mechanics are classically solved by the h-version of the finite element method [1]. The constraints are enforced along a priori defined interfaces on the surfaces of elastic bodies under consideration.

Contact material

We present a novel approach to model frictionless contact using high order finite elements (p-FEM) [2]. Here, a specially designed material is used, which is inserted into regions surrounding contacting bodies [3]. Contact constraints are thus enforced on the same manifold as the accompanying structural problem. Our contact material model is based on the hyperelastic formulation by Hencky [4]:

$$W_H(\lambda_1,\lambda_2,\lambda_3) = \mu \sum_{i=1}^{3} (\ln \lambda_i)^2 + \frac{\Lambda}{2}(\ln J)^2,$$

where

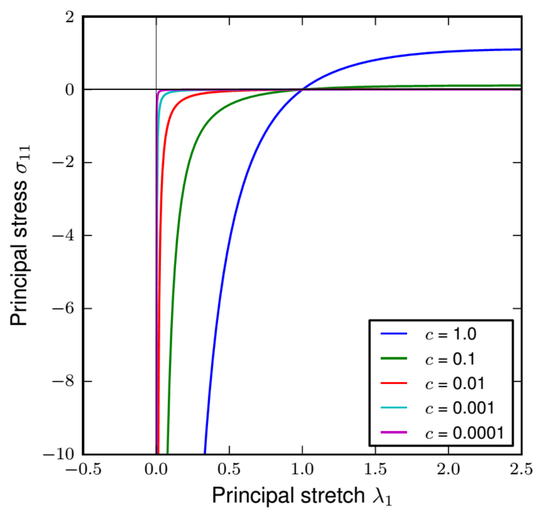
$$J = \lambda_1 \lambda_2 \lambda_3.$$

The material parameters μ and Λ are scaled by a contact stiffness c to regularize the Karush-Kuhn-Tucker conditions for normal contact:

$g \geq 0$	No normal penetration
$R \leq 0$	Only compressive forces
$g \cdot R$	Consistency.

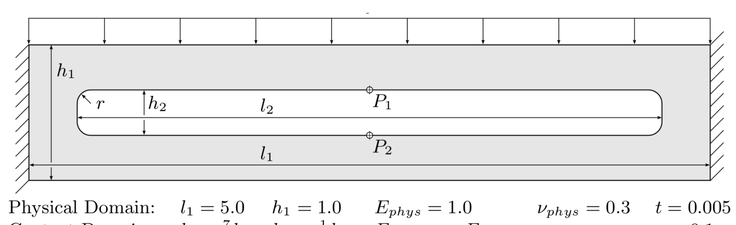
The resulting principal stresses then read

$$\sigma_{ii} = \frac{c}{J}\left(2\mu \ln(\lambda_i) + \Lambda \ln(J)\right)$$



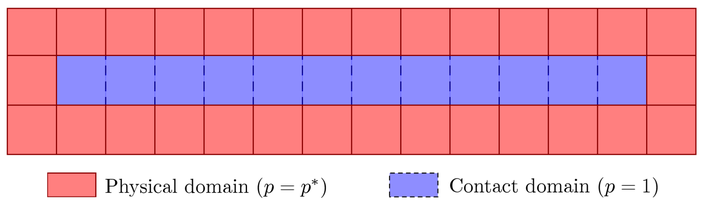
2D model problem

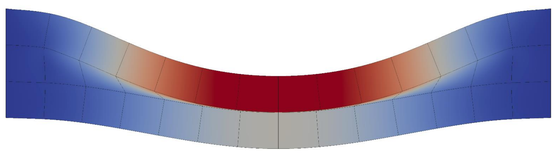
The model problem under consideration is a slotted block subjected to a constant, vertical load. The physical part shown in grey contains a neo-Hookean material, whereas the slot is filled with the contact material model. Fillets at the corners of the slot are treated according to the finite cell method [5].



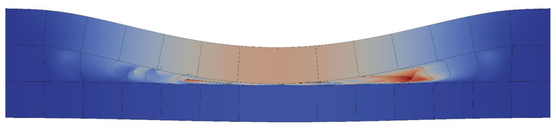
Physical Domain:	$l_1 = 5.0$	$h_1 = 1.0$	$E_{phys} = 1.0$	$\nu_{phys} = 0.3$	$t = 0.005$
Contact Domain:	$l_2 = \frac{7}{2} l_1$	$h_2 = \frac{1}{3} h_1$	$E_{fict} = c \cdot E_{phys}$	$\nu_{fict} = \nu_1$	$r = 0.1$

Numerical investigations showed, that modes inside the contact domain of order $p > 1$ might collapse. To overcome this problem, higher modes inside the contact domain are deactivated, while edge modes, on the interface to the physical domain remain active.



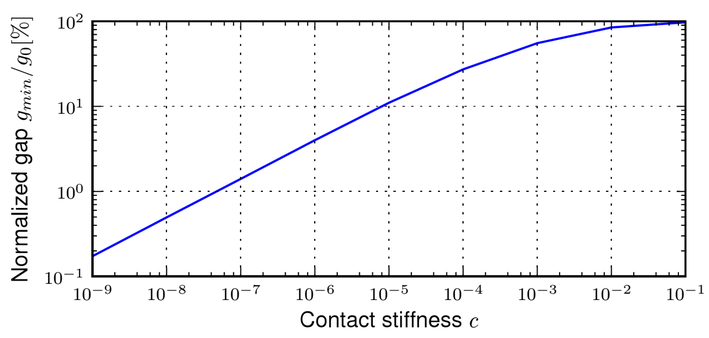


Stable solution in case of suppression of higher modes internal to the contact domain (p = 1).

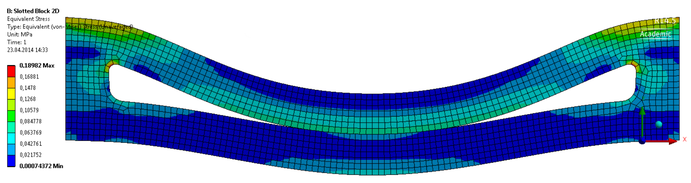


Instabilities in the contact domain in case of high order modes inside the contact domain (p = 4).

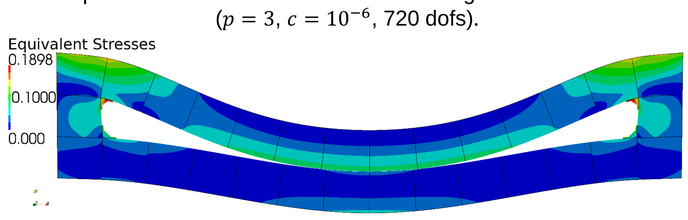
The influence of the contact stiffness c on the resulting minimum gap g_{min} is investigated for an ansatz order of $p = 3$. The ratio of g_{min} and the initial gap g_0 approaches zero as c is reduced. The material, thus, converges to the limit state defined by the KKT conditions. Furthermore, the gap ratio for a contact stiffness of $c = 10^{-5}$ already lies in the range of 10%, which is sufficient for many engineering applications.



The equivalent stress solution for an ansatz order $p = 3$ is compared to a simulation conducted with ANSYS, using quadratic elements. The results show similar stress distributions. However, the analysis using the contact material used significantly less degrees of freedom(720 dofs), than the ANSYS simulation (10,480 dofs).



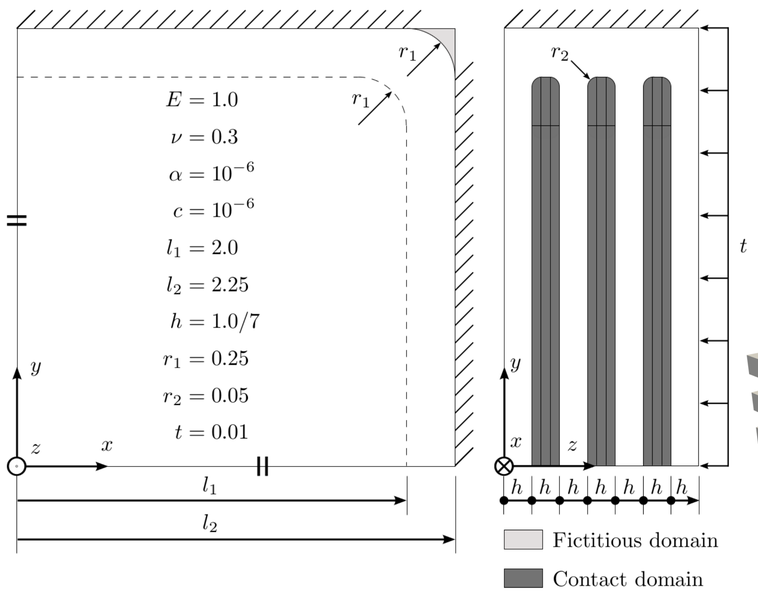
Equivalent von Mises stress obtained using contact material ($p = 3$, $c = 10^{-6}$, 720 dofs).

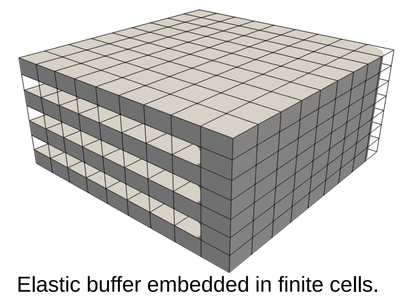


Equivalent von Mises stress obtained using ANSYS ($PLANE183$, 10.480 dofs) [6].

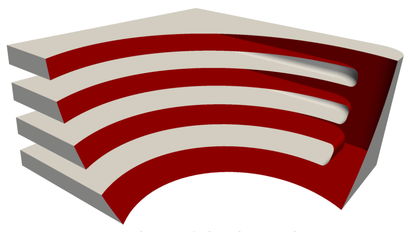
3D example: elastic buffer element

An elastic buffer element made up of several thin-walled layers is exposed to a distributed, vertical surface load. The geometry of the buffer element is embedded in a mesh of finite cells of ansatz order $p = 3$.

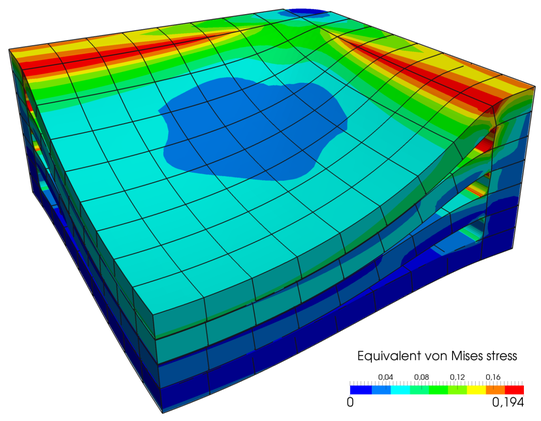




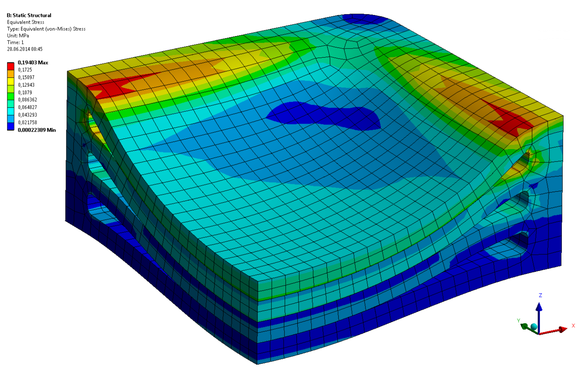
Elastic buffer embedded in finite cells.



Cut view of the boundary representation of the buffer.



Equivalent von Mises stress obtained using contact material ($p = 3$, $c = 10^{-6}$, 18,816 dofs).



Equivalent von Mises stress obtained using ANSYS ($SOLID186/7$, 89,124 dofs) [6].

Conclusion

The proposed formulation works well for non-matching discretizations on adjacent contact interfaces and handles self contact naturally. Since the non-penetrating conditions are solved in a physically consistent manner, there is no need for an explicit contact search. By application of high order finite elements, structures can be discretized with only a few coarse finite elements. This allows the simulation of complex deformation scenarios with a lower number of degrees of freedom, compared the h-version of the FEM.

References

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The financial support by the DFG under grant RA 624/15-2 is gratefully acknowledged.