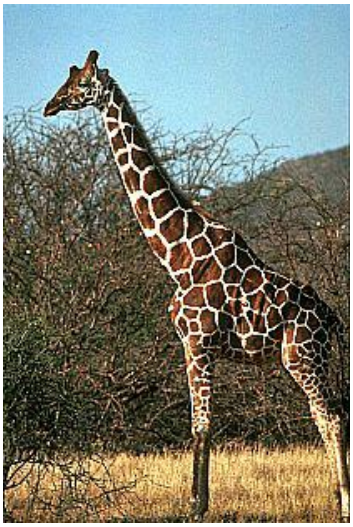
Topic 6:
Implementation of a Polyhedral Finite Element Formulation based on Voronoi Decomposition

The finite element method is a powerful computational method for the solution of differential equations which arise in various fields of engineering science. The key-idea is to subdivide the entire domain of a boundary value problem into sub-areas, the so-called finite elements, and to approximate inside an element the shape of the unknown field function in terms of specific ansatz functions governed by constant parameters. These parameters are then determined such that the differential equation is satisfied in a weighted-integral sense. For details on weighted residual methods we refer to *Zienkiewicz & Taylor* [1] and *Hughes* [2] among others.
A lot of different types of finite elements have been developed for a wide range of applications. Most formulations are based on decompositions in form of triangles or quadrilaterals. However, it is also possible to develop polyhedral finite elements by using Voronoi decomposition strategies, see e.g. *Milbradt* [3]. Here, a Voronoi diagram is a simple mathematical object that determines the nearest neighbour decomposition for a set of points in Euclidean space. Besides, nature provides numerous examples of Voronoi diagrams as the giraffe and the salt flat shown below.



Natural examples of polyhedral finite element meshes based on Voronoi diagrams

The aim of this software lab is a) to develop a formulation of hexahedral elements for heat transfer problems and b) to realize its implementation in a finite element code including the pre- and postprocessing steps. The project permits the decomposition in different modules that can be treated consecutively and therefore is well suitable for a software lab.

References

[1] O.C. Zienkiewicz, R. Taylor, *The Finite Element Method*, McGraw Hill, London, 1989.
[2] T.R.J. Hughes. *The Finite Element Method*, Oxford University Press, Prentice Hall, Englewood Cliffs, New Jersey, 1987.
[3] P. Milbradt. *Algorithmische Geometrie der Bauinformatik*, Habilitationsschrift am Institut für Bauinformatik der Universität Hannover, 2001.

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