Reliability-based optimization

Consider the optimal design of the cross section of the cantilever beam shown in Fig. 1. The mathematical definition of the corresponding optimization problem is given in Fig. 2a, with design variables $\mathbf{d} = [h, b]^{\mathrm{T}}$. In this case, the objective function $f(\mathbf{d})$ would be the the cross sectional area and the constraint $g(\mathbf{d}) \leq 0$ would require that the stresses at the support do not exceed the yield strength of the material y.

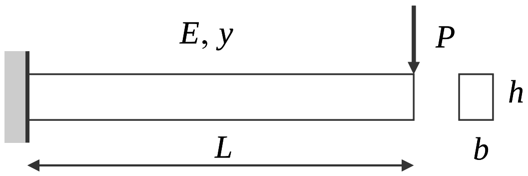

Fig. 1: Sizing optimization problem for a Cantilever beam

Consider now the case where we take into account the uncertainties in the model, i.e. the variability of the material parameters, the loading and the geometry. These uncertain parameters can be modelled by a vector of random variables $\mathbf{X}$ (for the cantilever example $\mathbf{X} = [P, E, y, L]^{\mathrm{T}}$). The corresponding reliability-based optimization problem is defined in its general form in Fig. 2b. In this case, we require that the probability that the constraint is satisfied is less than a given accepted probability value P_i^{accept}. The advantage of this approach is that it enhances the reliability of the design, since it ensures that the optimal design will satisfy the basic safety requirements.

$$\begin{aligned} &\text{minimize} && f(\mathbf{d}) \\ &\text{such that} && g_i(\mathbf{d}) \leq 0 \qquad i = 1, \ldots, N \\ & && \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned}$$

(a) Standard optimization problem

$$\begin{aligned} &\text{minimize} && f(\mathbf{d}) \\ &\text{such that} && P\left(g_i(\mathbf{d}, \mathbf{X}) \leq 0\right) \leq P_i^{accept} \quad i = 1, \ldots, N \\ & && \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned}$$

(b) Reliability-based optimization problem

Fig. 2: Definition of the standard and reliability-based optimization problems

In this software lab project, the students will become acquainted with the structural reliability theory [1], as well as with basic algorithms for reliability-based optimization (e.g. see [2]). They will subsequently implement suitable algorithms for the solution of the general reliability-based optimization problem and they will validate the performance of the algorithms by application to a number of optimization examples. These examples can be from any type of engineering application where reliability is relevant and will be selected jointly with the students.

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References

[1] R. Melchers, "Structural Reliability Analysis and Prediction", Second Edition, John Wiley & Sons, Chichester, 1999.
[2] G.I. Schuëller, H.A. Jensen, "Computational methods in optimization considering uncertainties", Computer Methods in Applied Mechanics and Engineering 198(1): 2-13, November, 2008.