

# Efficient Software for Computing Correlated K-Ss Tomographs

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# Computational Modeling of Saturated Flow

Governing differential equation:

$$\nabla \cdot (K \nabla h) + S_s \dot{h} = q$$

$K = K(x, y, z)$  = hydraulic conductivity

$S_s = S_s(x, y, z)$  = specific storage

$h = h(x, y, z, t)$  = potentiometric head

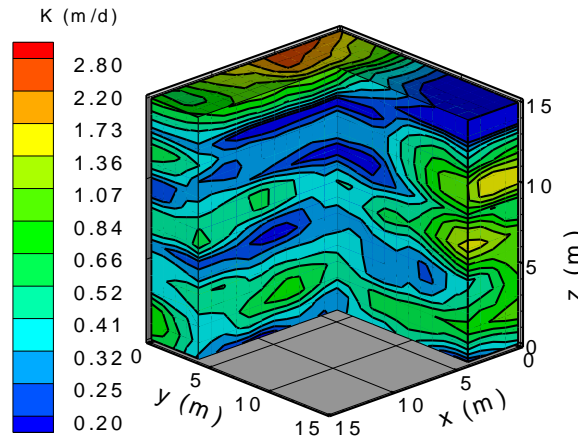
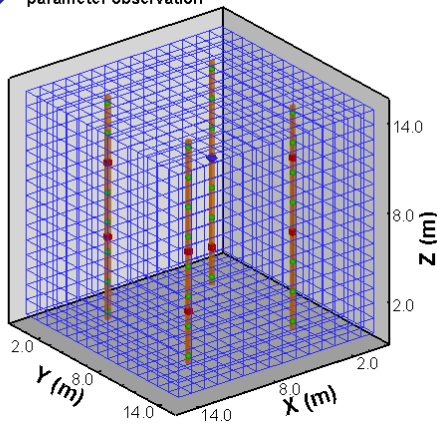
$q = q(x, y, z, t)$  = source/sink rate

Finite element solution:

$$[K] \{h\} + [S_s] \{\dot{h}\} = \{q\}$$

**Hydraulic Tomography –**  
*‘CAT scan’ of the subsurface*

- pumping location
- head observation
- parameter observation



Yeh and Liu (2000) to estimate the spatial distributions of K and  $S_s$  (tomographs/images) by applying hydraulic stresses at various locations sequentially and observing the hydraulic responses at other measurement locations

(From Professor Jim Yeh at University of Arizona)

# Software Lab Project Tasks

In this software lab project, the students will:

- (1) develop a program to compute the hydraulic responses ( $h$ ) to hydraulic stresses ( $q$ ) by using linear 3D finite elements to solve the governing differential equation;
- (2) implement the adjoint sensitivity method to efficiently compute the first-derivatives of  $h$  with respect to  $K$  and  $S_s$  at each pixel;
- (3) test the significance of incorporating  $K$ - $S_s$  correlation on the tomographs using data at University of Waterloo experimental site.

*(Optional) – If time allows, implement efficient representations of correlated  $K$ - $S_s$  fields to obtain compressed high-resolution tomographs for large scale problems.*