

## Software Lab:

Mechanics: ★★☆☆☆  
Mathematics: ★★★★★  
Programming: ★★☆☆☆

# Virtual Element Method

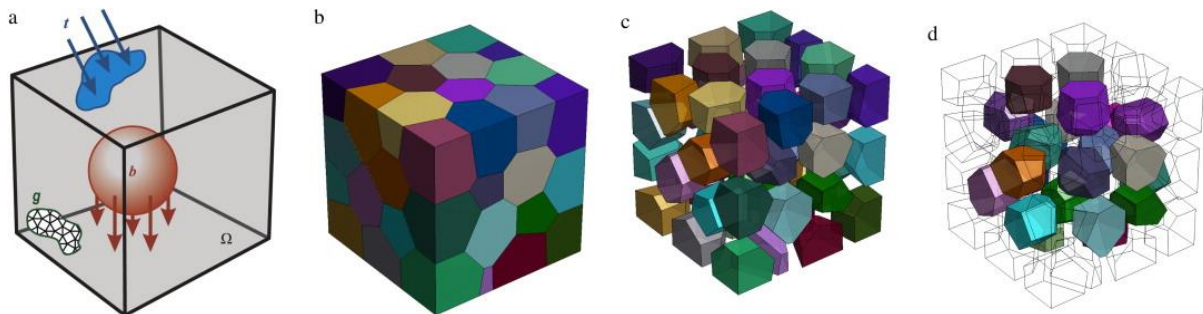
## Setting

The finite element method integrates the weak form of differential equations on reference elements. These are either triangles or quadrilaterals in 2D. In 3D the user's choice is usually limited to tetrahedra or hexahedra. Recently, the *virtual element method* was proposed by Brezzi et al. [1] which allows for arbitrarily shaped polyhedrons in 3 dimensions. How does this method work? What are its advantages / disadvantages as compared to standard FEM and, most importantly: how can it be programmed?

## Task

Your task is manifold:

- Carry out a literature-research on the method and dig through its mathematical foundations.
- Program the method in 2D and 3D for linear elasticity and evaluate its performance on benchmarks.
- Consider extensions to high-order schemes and non-linear elasticity



(a) Schematic illustrating the elasticity boundary value problem. (b) Partition of the domain. (c) Split view of the discretized domain. (d) View of a few boundary and internal elements.

Graphic taken from [2]

## Supervisors

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## References

- [1] L. Beirão Da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L.D. Marini, A. Russo, Basic principles of Virtual Element Methods, *Math. Models Methods Appl. Sci.*, 23 (1) (2013), pp. 199–214
- [2] A. L. Gain, C. Talischi, G. H. Paulino, On the Virtual Element Method for three-dimensional linear elasticity problems on arbitrary polyhedral meshes, *Computer Methods in Applied Mechanics and Engineering*, Volume 282, 1 December 2014, Pages 132-160